6.3 Binomial and Geometric **Random Variables**

**Binomial Settings**

A **binomial setting** arises when we perform several independent trials of the same chance process and record the number of times that a particular outcome occurs. The four conditions for a binomial setting are

1. **B**inary? The possible outcomes of each trial can be classified as “success” or “failure.”
2. **I**ndependent?Trials must be independent; that is, knowing the result of one trial must not have any effect on the result of any other trial.
3. **N**umber?The number of trials *n* of the chance process must be fixed in advance.
4. **S**uccess?On each trial, the probability *p* of success must be the same.

The count *X* of successes in a binomial setting is a **binomial random variable.** The probability distribution of *X* is a **binomial distribution** with parameters *n* and *p*, where *n* is the number of trials of the chance process and *p* is the probability of a success on any one trial. The possible values of *X* are the whole numbers from 0 to *n*.

Example:

Each child of a particular pair of parents has probability 0.25 of having type O blood. Genetics says that children receive genes from each of their parents independently. If these parents have 5 children, the count *X* of children with type O blood is a binomial random variable with *n* = 5 trials and probability *p* = 0.25 of a success on each trial. In this setting, a child with type O blood is a “success” (S) and a child with another blood type is a “failure” (F).

What’s *P(X* = 2)?

**Binomial Coefficient**

The number of ways of arranging *k* successes among *n* observations is given by the **binomial coefficient**



where *n*! = *n*(*n –* 1)(*n* – 2)*•…•*(3)(2)(1) and 0! = 1.

**Binomial Probability**

If *X* has the binomial distribution with *n* trials and probability *p* of success on each trial, the possible values of *X* are 0, 1, 2, …, *n*. If *k* is any one of these values,



Example: Inheriting Blood Type

Each child of a particular pair of parents has probability 0.25 of having blood type O. Suppose the parents have 5 children

**(a) Find the probability that exactly 3 of the children have type O blood.**

1. **Should the parents be surprised if more than 3 of their children have type O blood?**

Example: Telephone Connections

In the old days, when operators had to manually connect the calls, there was a probability of 0.8 of success in any attempt to make a telephone call. (This often depended on the importance of the person making the call, or the operator's curiosity!)

1. **Calculate the probability of having 7 successes in 10 attempts.**
2. **What is the probability that you will get at least 8 successful calls in 10 attempts?**

**Using the Calculator for Binomial Probabilities**

**For the probability of a single value**

Menu --- 5 Probability --- 5 Distributions --- D Binomial PDF

Fill out all three boxes and hit OK

**For the probability of multiple values**

Menu --- 5 Probability --- 5 Distributions --- E Binomial CDF

Fill out all four boxes and hit OK

* If a “less than situation”, Lower Bound is 0
* If a “more than situation”, Upper Bound is n